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Local Modulus of Elasticity-- Intuition and Ramifications

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Abstract

We propose software changes to lumber stress rating machinery that will result in more accurate sorting and increased market value for the lumber produced. Bending modulus of elasticity (E) measurement of lumber is useful in rapidly evaluating the structural value of each piece. Typically, a production machine obtains a sequence of measurements on overlapping spans along a piece of lumber at lineal speeds to 18 m/s. In one type of low-speed machine, these measurements occur every 150 mm on spans of 900 mm; in a high-speed machine, they occur every 14 mm on spans of 1220 mm. While E in lumber is important by itself for determining the stiffness of floor systems and the strength of long thin columns, it correlates also with tensile and bending strengths. Consequently, E is used to help categorize each piece of lumber into a structural grade. Evidence exists that improved correlation with strength, and hence better sorting for value, would be obtained if bending E were available on much shorter spans. However, a direct bending measurement of "local E" is impractical. Instead, we estimate local E with method guided by our intuition about how redundancy from a sequence of overlapping bending measurements and correlation of property values along the piece length may be used. We present results for local E from both a Kalman filter and, separately, from solutions of a linear system of equations that describe the measurement. We have gained confidence in the methods because they lead to similar local E values. Each of the methods can be implemented with software changes and minimal disruption of production and quality control processes.

Introduction and Objectives

There is good reason to believe that bending E from very short spans would correlate better with strength than E from the approximately one meter spans now used in production of machine stress rated lumber (Kass 1975, Orosz 1976). Research has shown that lumber failures in bending or in tension begin usually at a local characteristic, such as grain deviation, that would show up as reduced E at that location. But it is impractical to make bending tests on very short spans, and the resolution of the measurement suffers from any reduction in length. Instead, we estimate E on very short spans. Imagine that a piece of lumber has been subdivided into contiguous equal length increments. Our technical objective is to estimate E, denoted as local E, in every increment of this subdivision where increment lengths are about 50 mm or less. Our goal is to increase market values of tested lumber populations. Previous work (Bechtel 1985, Foschi 1987, Lam et al. 1993, Pope and Matthews 1995) also studied local E. That work was valid only for simply supported bending spans and had other practical problems addressed elsewhere (Bechtel et al. 2006, 2007a, 2007b).

The previous work has shown that it is advantageous to work with local compliance C defined as the reciprocal of local E and with measured compliance C_M defined similarly; see Equations **[1]**. We also include variously the cross-sectional moment of inertia I in the definition of C and C_M, i.e. $C = 1/(EI)$ and $C_M = 1/(E_M I_M)$ depending on whether we want to consider I as varying along the length.

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$$C = \frac{1}{E} \quad , \quad C_M = \frac{1}{E_M} \quad ,$$ subscript "M" distinguishes measured from local values. **[1]**

Measurement Redundancy and Correlation Among Subdivision Increments

While we are unable to measure local C of a short increment directly by a bending test, this local C contributes to every measurement C_M in a sequence of overlapping measurements on longer bending spans, each of which includes the increment of interest. This is a type of measurement redundancy, and intuitively, each of these measurements should be useful in estimating local C for that increment. Further, two closely spaced points in a piece of lumber come from closely spaced points within a tree. Intuitively, the correlation of local C for these increments should be greater than for increments farther apart. Given local C for an increment, statistically, our uncertainty about local C for its neighbors is reduced greatly. This correlation should be useful in estimating local C for each increment.

Span function

A bending measurement involves a bending span defined by a system of supports and a means for applying and/or measuring force and deflection. From the configuration of the bending span and the size of the tested piece, we obtain measured compliance C_M. This number represents compliance within the bending span, and all of the local compliance values within the bending span influence the measured result C_M. Span function is a weighting function that describes how much the compliance C at each point along the bending span contributes to C_M. A method of computing the span function for a general system of supports is available (Bechtel et al. 2006, Bechtel 2007). **Figure 2** illustrates the span function for the 900 mm, simply-supported, center-loaded bending span of **Figure 1**. From **Figure 2**, the compliance at span center contributes most to the measurement, the compliance at span ends contributes nothing, and the compliance halfway from an end to center span contributes one-fourth as much as the compliance at span center. The total area under the curve is one; it must be for a valid weighting function.

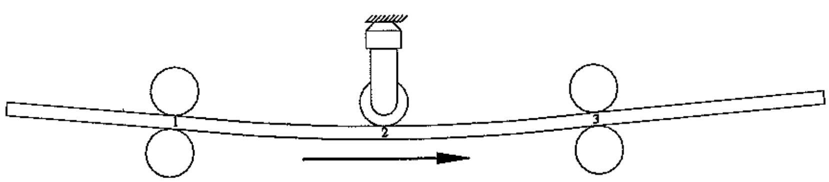

Figure 1.—*Mechanical schematic of a simply supported, center-loaded bending span.*

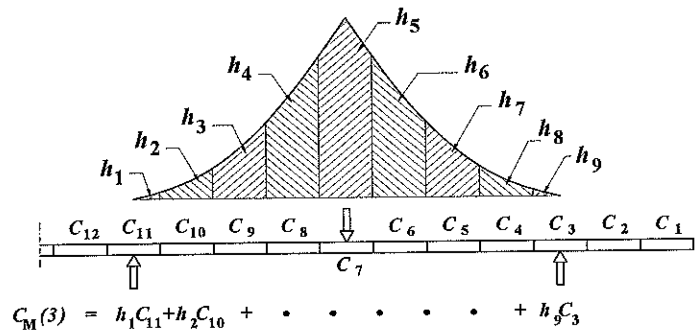

Figure 2.—*Span function for simply-supported, center-loaded bending span. Span weights $h_1, h_2, \ldots, h_9$ are shown applied to local compliances to obtain measured compliance C_M.*

In **Figure 2**, span weights $h_1, h_2, \ldots, h_9$ are areas under the span function in regions corresponding to increments of the piece subdivision. These weights are applied to local compliances in the subdivision as shown to obtain our model of measured compliance $C_M(3)$ as a weighted moving average of the local compliances. If the piece is moving to the right, the next measurement after one increment of motion, is: $C_M(4) = h_1C_{12} + h_2C_{11} + \ldots + h_9C_4$. The coarse subdivision of **Figure 2** simplifies illustration; normally, there would be many more increments in the subdivision over the span length.

Another type of machine uses a series of clamp rollers at span ends to isolate bending measurements from dynamic effects of transverse lumber motion outside the bending span. The support arrangement for a bending span of this machine and two span functions for it are illustrated in **Figure 3** and **Figure 4**, respectively. For a piece long enough to engage all supports of the bending span, i.e. longer than **14** in **Figure 3**, there are five different span functions for which we can make measurements. This is because the leading end **7** of the piece, as it progresses rightward through the machine, first engages only one support at $x_5 = 610$ mm at the right end of the span, then two supports, and then three. As the piece continues rightward, its trailing end **10** engages three, then two, and finally just one support at $x_3 = -610$ mm at the left end of the span. The dashed curve in **Figure 4** is the span function applicable when only the first support at the right end of the span is engaged as in **Figure 3**. The solid curve is applicable when the piece engages all seven supports.

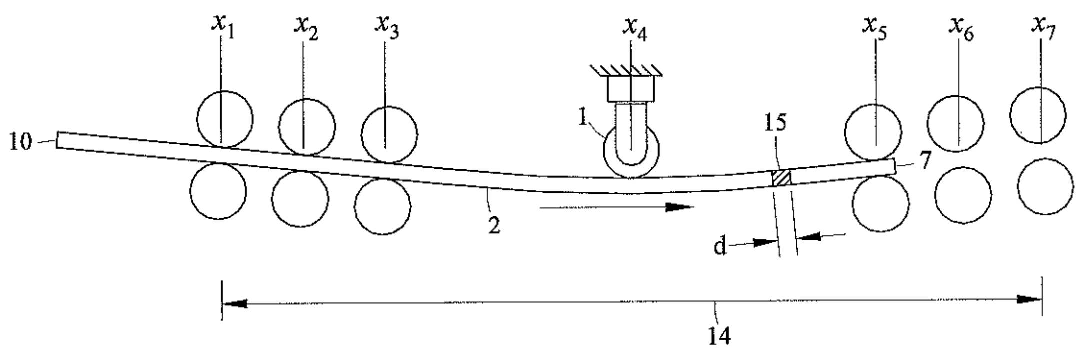

Figure 3.—*Mechanical schematic of a clamp-roller-supported, center-loaded bending span*

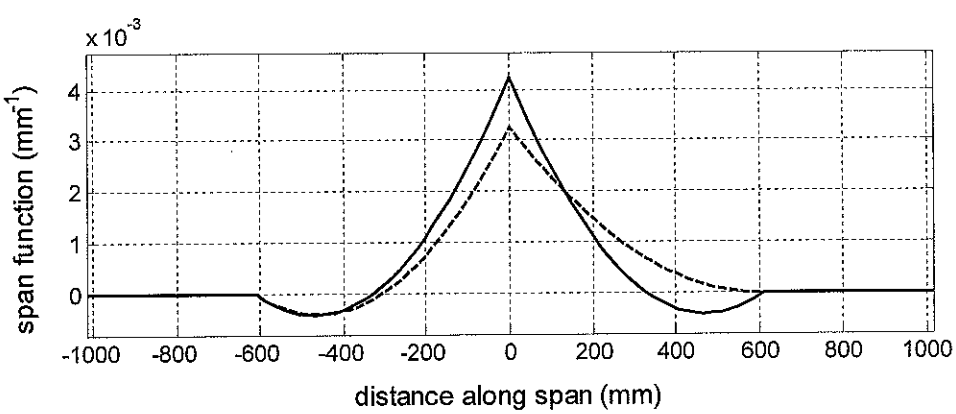

Figure 4.—*Two of the span functions for the clamp-roller-supported span of Figure 3. The dashed line corresponds to the piece of lumber as in Figure 3 engaging just one support at the right end of the span. The solid line applies to the case when all supports are engaged.*

Figure 5 shows, on the same scale, span functions for the two types of bending spans in **Figure 1** and **Figure 3**. We see that even with its longer (1220 vs. 900 mm) span, the clamp-roller-supported span has a sharper peak and thus better resolution along the piece length than the simply-supported span. Span functions are useful for machine design purposes.

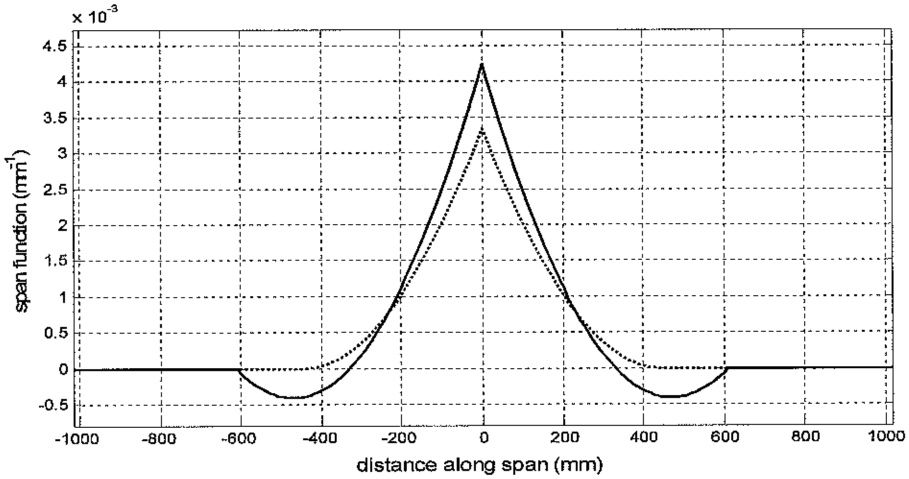

Figure 5.—*Span functions for the 1220 mm span of Figure 3 when all supports are engaged (solid line), and for the 900 mm span of Figure 1 (dotted line).*

Test Piece and Test Apparatus

The compliance C_M of one piece of Douglas Fir lumber of size 38 mm x 89 mm x 4500 mm was measured on a sequence of 73 overlapping bending spans spaced 50 mm apart along the piece length. This piece was selected as being well-machined and for its interesting knot characteristics. Simply-supported, center-loaded bending test apparatus was used with a fixed load increment of 565 N. Deflection measurement resolution was 0.01 mm. We can use the same methods for other more complicated test apparatus, but span functions appropriate to that geometry would be used.

Overview and Results of Local Compliance Estimation by Kalman Filtering

Figure 2 and the accompanying discussion show that the sequence of compliance measurements is a moving weighted average of local compliance values (discrete convolution), where the weights come from a span function. This allows our model to account for measurement redundancy, whereby each local compliance can be a component of more than one measurement. We account for *a priori* knowledge of correlation among local compliances by modeling them as an autoregressive random process. Combining these concepts, we model the compliance measurement as the output of a linear dynamic system, this output being an autoregressive, moving average (ARMA) random process, thereby allowing a state-space description (e.g. Ogata 1987). With this format, we have a state vector describing the state of the system, and an output. The state vector has component state variables that are local compliances within the bending span, and the output is a weighted linear combination of the state variables. A Kalman filter (Kalman 1960) operates on the measured compliance sequence to estimate the state vector, i.e. local compliances. The Kalman estimates are optimal in the mean square error sense.

This overview is grossly incomplete in its detail. Details are available in the literature (Bechtel et al. 2006, 2007a, 2007b) although the 2006 patent as printed contains numerous errors. For those interested in the patent, be certain to obtain either the errata for it or to ask for the patent specification as it was submitted to the patent office. The author can provide assistance as necessary.

Figure 6 illustrates estimated local E when a correlation coefficient in the model is set to 0.98. The dashed line is the measured E_M, and the solid line is the estimated local E from the Kalman filter. The dotted line labeled "10 x error" is 10 times the difference between the measured E_M and the result obtained by passing the local compliance estimates mathematically back through the machine model, making use of the span weights. We state an error measure in percent that relates this error to the measurement. For better visualization, we have scaled the plotted error up by a factor of ten.

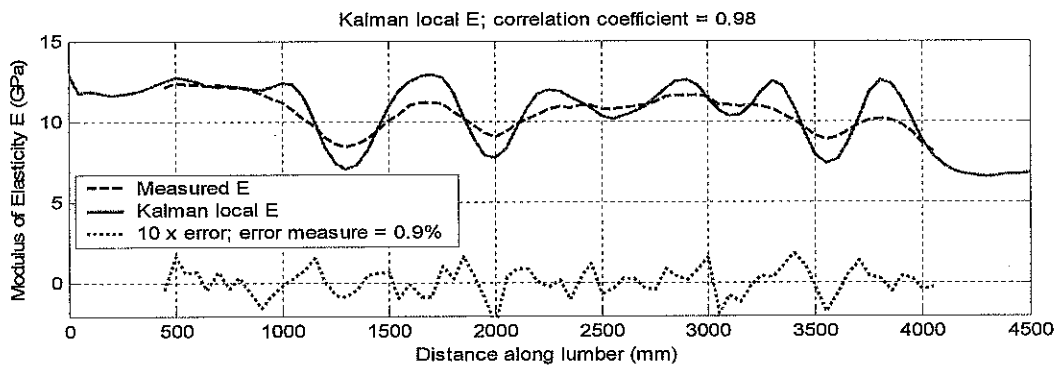

Figure 6.—*Kalman estimated local E. Correlation coefficient = 0.98.*

We observed serious knots on the tested piece corresponding to the pronounced dips in the local E curve at approximately 1300 mm, 2000 mm, 3550 mm and 4150 mm from the leading end of the piece. While we see the effect of the knots on the measured curve as well, we can assess their severity better by using the estimated local E. We observed lesser knots at 2350 mm and 3100 mm and knot-affected grain deviations from about 2500 mm to 2750 mm.

Local *E* by Solution of Linear Equations

Referring back to **Figure 2** and accompanying discussion, we can write each of the m measured compliances as a linear combination of the n local compliances. The resulting equations are:

$$\begin{aligned} h(p)C(1) + h(p-1)C(2) + \cdots + h(1)C(p) &= C_M(1) \\ h(p)C(2) + h(p-1)C(3) + \cdots + h(1)C(p+1) &= C_M(2) \\ &\vdots \\ h(p)C(n-p+1) + h(p-1)C(n-p+2) + \cdots + h(1)C(n) &= C_M(m) \end{aligned} \qquad \textbf{[2]}$$

There are m equations in n unknowns, which is an "underdetermined" linear system of equations with more unknowns (the n local compliances) than equations. We can represent Equations **[2]** in matrix form as $HC = C_M$, where H is a "span matrix" consisting of span weights. C and C_M are vectors of local compliances and measured compliances respectively. For our tests with the test piece, there are $n = 91$ unknown local compliances and $m = 73$ known measured compliances. There does not have to be an exact solution, but in this case, because span matrix H has rank m, there is a whole family of exact solutions. We can introduce additional criteria or constraints to help select the solution we want.

Minimize sum of squares of computed local compliances

Figure 7 shows the result of solving the system of Equations **[2]** with the additional constraint that the sum of the squares of the computed local compliances is minimized. The result of **Figure 7** is noisy, so we smoothed the measured C_M by filtering before using Equations **[2]** to obtain the local E of **Figure 8**. Filtering consisted of a weighted average of each C_M with its two nearest neighbors on each side except at the ends where we truncated the weights. The weights came from the fifth row of Pascal's triangle (1, 4, 6, 4, 1), and we scaled these to add to one. This smoothing reduces measurement noise that is always a component of the signal. The local E of **Figure 8** is smoother than that of **Figure 7** and is more reasonable. When we pass the local compliance C mathematically back through the system, using the span weights as in Equations **[2]**, we obtain computed measured C_M, which is just the filtered C_M. The error is the difference between measured C_M and its filtered version because solutions of Equations **[2]** return its right-hand side exactly.

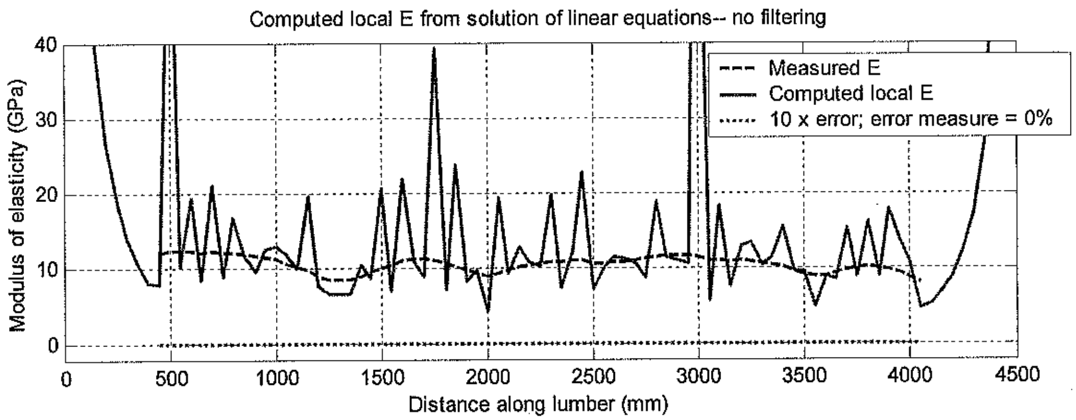

Figure 7.—*Measured E_M and computed local $E = 1/C$ from Equations [2].*

At 1%, the error in **Figure 8** is closer to what we would expect from compliance measurements. Because of our choice from among the family of solutions, the computed local compliances near the ends are very small. The corresponding computed local *E* values, therefore, are very large in both **Figure 7** and **Figure 8**.

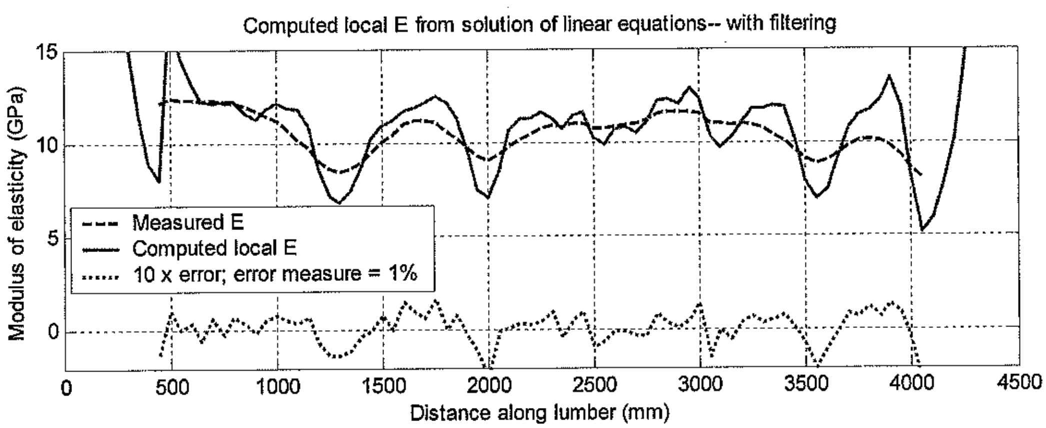

Figure 8.—*Measured E_M and computed local $E = 1/C$. Measured compliance C_M was filtered before solving Equations [2]. Note local E rising near piece ends.*

Proprietary Choice from among the Family of Solutions to Equations [2]

We now choose a solution to Equations **[2]** that makes more sense for our problem and results in **Figure 9** and **Figure 10**. In **Figure 9**, the same filtering was used as in **Figure 8**. In **Figure 10**, the filtering was more extensive. This solution utilizes correlation coefficient between adjacent local compliances; however, the results are insensitive for a wide range of correlation coefficient values (here 0.98). Note that computed local E follows measured E_M closely in **Figure 10**, but the error between computed and actual measurements is an unreasonably large 3.8 percent. In **Figure 10**, 1/(filtered C_M) is plotted also. The error is the difference between measured E_M and 1/(filtered C_M).

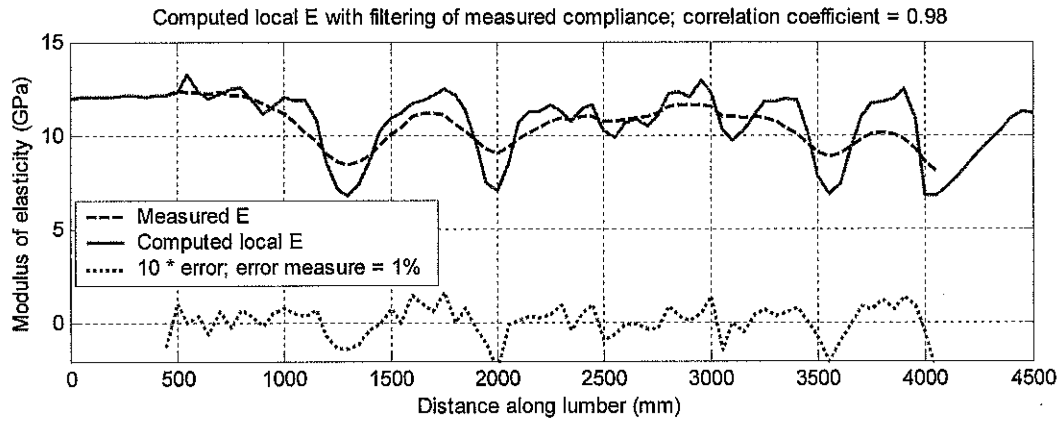

Figure 9.—*Measured E_M and computed local E from proprietary method.*

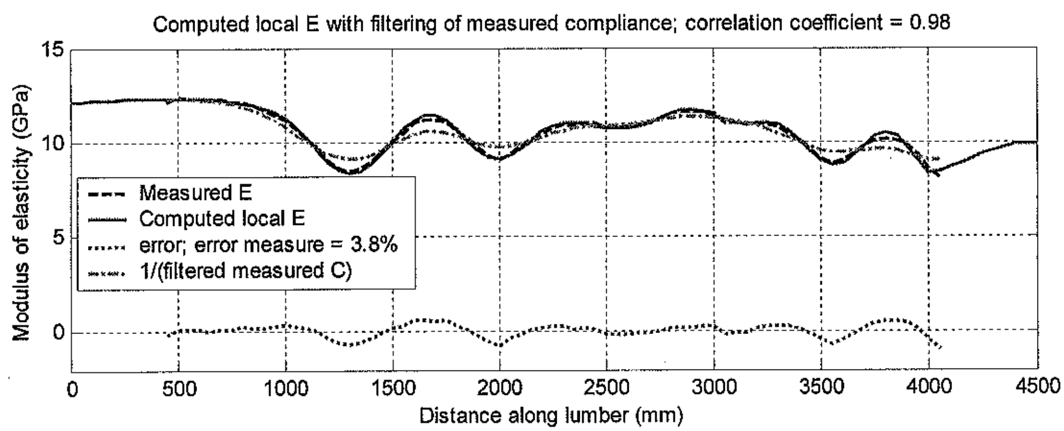

Figure 10.—*Measured E_M and computed local E from proprietary method. Similar to Figure 9, but with extensive filtering applied to measured compliance C_M.*

Implementation

While only a software change is necessary for implementation of the method in production equipment, off-line quality control (QC) issues must be addressed. We suggest first using a form of local E estimation (Kalman filter parameters also can be adjusted to give local E estimates that closely follow measured E) such as that resulting in **Figure 10**, because the computed local E is similar to the presently measured E, but without the noise that is often a component of the measured signal. Reduced noise alone will cause sorting improvement by eliminating downgrades resulting from noise. Existing grade thresholds should require only minimal if any adjustment, and QC would proceed without change. After gaining experience with the new software, we suggest reducing the extent of measured compliance filtering while simultaneously reducing machine thresholds as existing QC procedures allow. The objective is to downgrade those pieces that will fail the off-line QC tests, yet accept presently downgraded pieces that have sufficient strength and *E* to pass the QC tests. Evidence from the figures here, especially when compared to the tested piece of lumber, suggest that **Figure 9** portrays local *E* more accurately than **Figure 10**. We want to reach the point where we can use the local *E* as in **Figure 6** or in **Figure 9** to sort pieces for strength. We expect this more accurate sorting for strength to cause an overall increase in identified structural value and market value of the tested lumber product mix.

Conclusions and Ramifications

With only a software change, we predict increased market value for lumber from producers that use stress rating machinery. The change will cause more accurate sorting for strength and lead to greater confidence in the structural value of each piece tested. Implementation of the software change in the mill can be accomplished smoothly with minimal disruption to the grading process. That similar results are obtained with more than one approach for obtaining local *E* gives us confidence in the results. It is now time to implement the ideas in a production facility to prove the anticipated benefits.

Those interested in machinery design can use the span function concept to analyze proposed bending test span designs and performance before committing the designs to hardware.

References

Bechtel, F.K. 1985. Beam stiffness as a function of pointwise E, with application to machine stress rating. Proc. Int'l Symp. on Forest Products Research. CSIR. Pretoria, South Africa

Bechtel, F.K. 2007. Estimating local compliance in a beam from bending measurements; Part I. Computing "span function". Wood Fiber Sci. 39(2):250-259

Bechtel, F.K., C.S. Hsu, and T.C. Hanshaw. 2006. Method for estimating compliance at points along a beam from bending measurements. U.S. Patent No. 7,047,156

Bechtel, F.K., C.S. Hsu, and T.C. Hanshaw. 2007a. Estimating local modulus of elasticity in a beam from bending measurements, an overview. Forest Products Journal. 57(1/2):118-126

Bechtel, F.K., C.S. Hsu, and T.C. Hanshaw. 2007b. Estimating local compliance in a beam from bending measurements-- Part II: Optimal estimation of local compliance. Wood Fiber Sci. 39(2):260-270

Foschi, R.O. 1987. A procedure for the determination of localized modulus of elasticity. Holz als Roh-und Werkstoff. 45:257-260

Kalman, R.E. 1960. A new approach to linear filtering and prediction problems. Trans. ASME Journal Basic Engineering. Series 82D. pp. 35-45.

Kass, A.J. 1975. Middle ordinate method measures stiffness variation within pieces of lumber. Forest Products Journal. 25(3):33-41

Lam, F., R.O. Foschi, J.D. Barrett, and Q.Y. He. 1993. Modified algorithm to determine localized modulus of elasticity of lumber. Wood Science and Technology. 27:81-94.

Ogata K. 1987. Discrete-Time Control Systems. Prentice Hall, Englewood Cliffs NJ. 994 pp

Orosz, I. 1976. Relationship between apparent modulus of elasticity, gage length, and tensile strength of lumber. Wood Science and Technology. 10. pp. 273-291.

Pope, D.J. and F.W. Matthews. 1995. A comparison of deconvolution techniques to improve MOR estimation from stress grading machine output. Wood Science and Technology 29. pp. 431-439